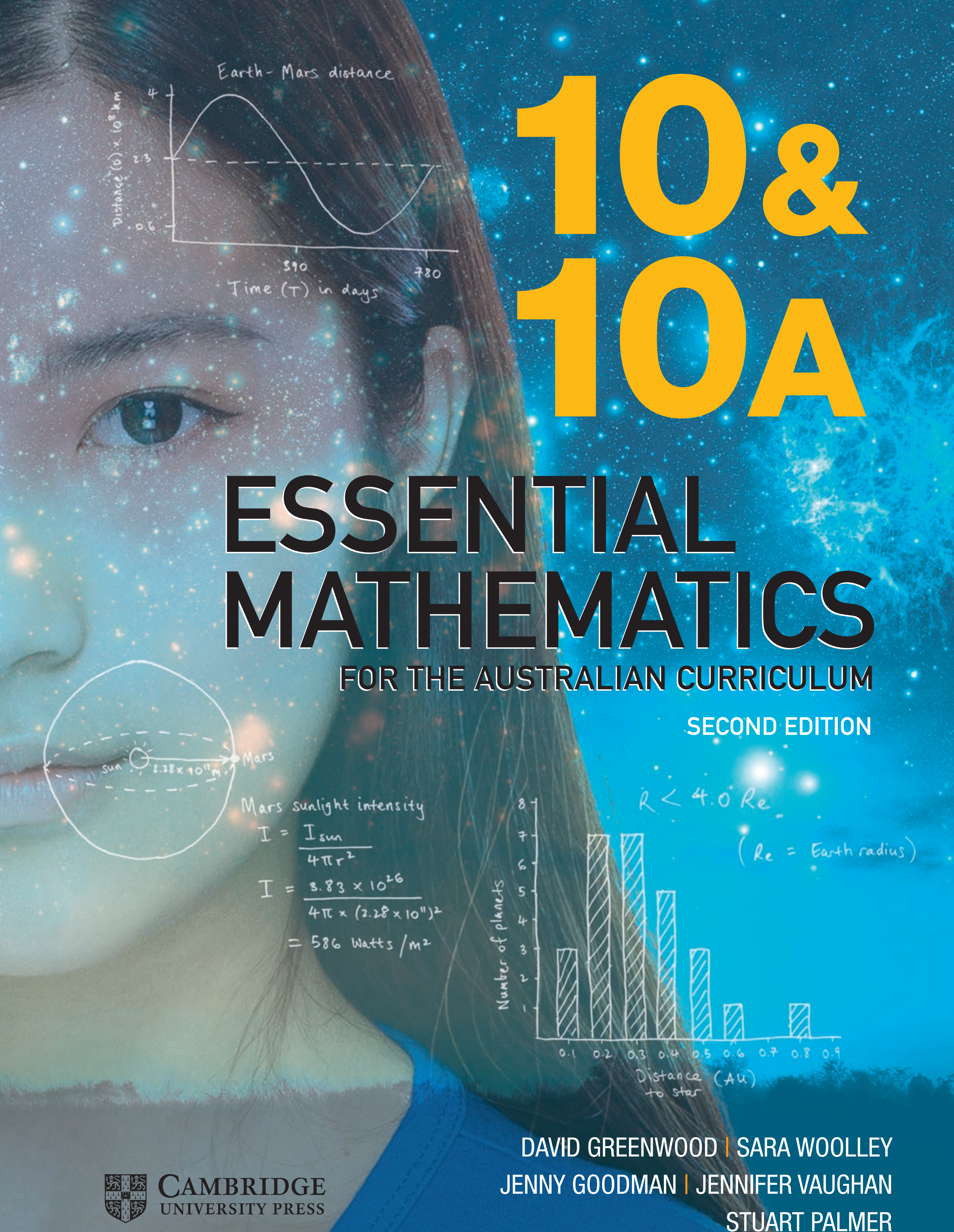

10 & 10A

ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM

SECOND EDITION

DAVID GREENWOOD | SARA WOOLLEY
JENNY GOODMAN | JENNIFER VAUGHAN
STUART PALMER

CAMBRIDGE UNIVERSITY PRESS

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About the authors

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 21 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006, has written more than 10 mathematics titles and specialises in lesson design and differentiation.

Jennifer Vaughan has taught secondary mathematics for over 30 years in New South Wales, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in New South Wales and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Stuart Palmer has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over New South Wales and beyond. He is a Project Officer with the Mathematical Association of New South Wales, and also works with pre-service teachers at The University of Sydney and The University of Western Sydney.

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Cambridge University Press

Introduction

This second edition of *Essential Mathematics for the Australian Curriculum* has been developed into a complete resources pack comprising a revised and updated print textbook, a new interactive textbook with a host of cutting-edge features, and an online teaching suite.

The successful elements of the first edition have been retained and improved upon. These include:

- logical sequencing of chapters and development of topics
- careful structuring of exercises according to the four Australian Curriculum proficiency strands plus enrichment
- graduated difficulty of exercise questions within the overall exercise and within proficiency groups
- *Let's Start* and *Key Ideas* to help introduce concepts and key skills.

Additions and revisions to the text include:

- New topics reflecting updates to the Australian Curriculum and state syllabuses
- Revision and extension topics are marked as 'Consolidating' or 'Extending' to help customise the course to each classroom's needs
- The working programs have been subtly embedded in each exercise to differentiate three student pathways: Foundation, Standard and Advanced
- A 'Progress quiz' has been added about two-thirds of the way into each chapter, allowing students to check and consolidate their learning – in time to address misunderstandings or weaknesses prior to completing the chapter
- Pre-tests have been revised and moved to the interactive textbook.

Features of the all-new interactive textbook:

- Seamlessly blended with Cambridge HOTmaths, allowing enhanced learning opportunities in blended classrooms, revision of previous years' work, and access to *Scorcher*
- Every worked example in the book is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'
- A searchable dictionary of mathematical terms and pop-up definitions in the text
- Hundreds of interactive widgets, walkthroughs and games
- Automatically-marked quizzes and assessment tests, with saved scores
- Printable worksheets (HOTsheets) suitable for homework or class group work.

Features of the Online Teaching Suite, also powered by Cambridge HOTmaths:

- A test generator, with ready-made tests
- Printable worked solutions for all questions
- A powerful learning management system with task-setting, progress-tracking and reporting functions.

The chart on the next pages shows how the components of this resource are integrated.

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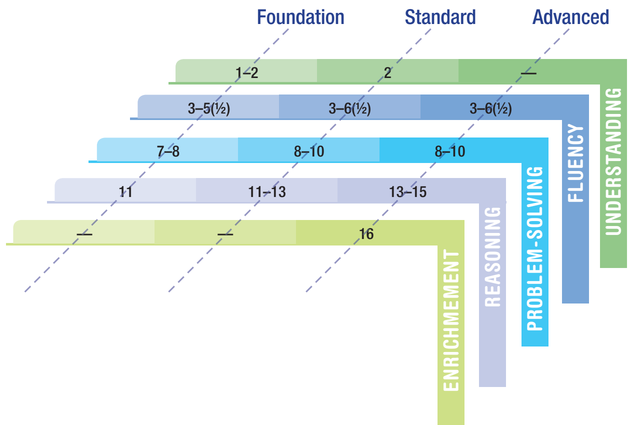
Guide to the working programs in the exercises

The working programs that were previously available in separate supporting documents have been updated, refined and embedded in the exercises for this second edition of *Essential Mathematics for the Australian Curriculum*. The suggested working programs provide three pathways through the course to allow differentiation for Foundation, Standard and Advanced students.

As with the first edition, each exercise is structured in subsections that match the four Australian Curriculum proficiency strands (Understanding, Fluency, Problem-solving and Reasoning) as well as Enrichment (Challenge). The questions suggested for each pathway are listed in three columns at the top of each subsection:



- The left column (lightest shaded colour) is the Foundation pathway
- The middle column (medium shaded colour) is the Standard pathway
- The right column (darkest shaded colour) is the Advanced pathway.

Gradients within exercises and proficiency strands

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise – where there is an increasing level of mathematical sophistication required from Understanding through to Reasoning and Enrichment – but also within each proficiency strand; the first few questions in Fluency, for example, are easier than the last few, and the last Problem-solving question is more challenging than the first Problem-solving question.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Foundation pathway will likely need more practice at Understanding and Fluency, but should also attempt the easier Problem-solving and Reasoning questions. An Advanced student will likely be able to skip the Understanding questions, proceed through the Fluency questions (often half of each question), focus on the Problem-solving and Reasoning questions, and attempt the Enrichment question. A Standard student would do a mix of everything.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them.
If required, the chapter pre-tests (now found online) can also be used as a diagnostic tool. The following are recommended guidelines:

- A student who gets 40% or lower in the pre-test should complete the Foundation questions
- A student who gets between 40% and 85% in the pre-test should complete the Standard questions
- A student who gets 85% or higher in the pre-test should complete the Advanced questions.

For schools that have classes grouped according to ability, teachers may wish to set one of the Foundation, Standard or Advanced pathways as their default setting for their entire class and then make individual alterations depending on student need. For schools that have mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

* The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1-4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e, ... or b, d, f, ...)
- 2-4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- — : do not complete any of the questions in this section.

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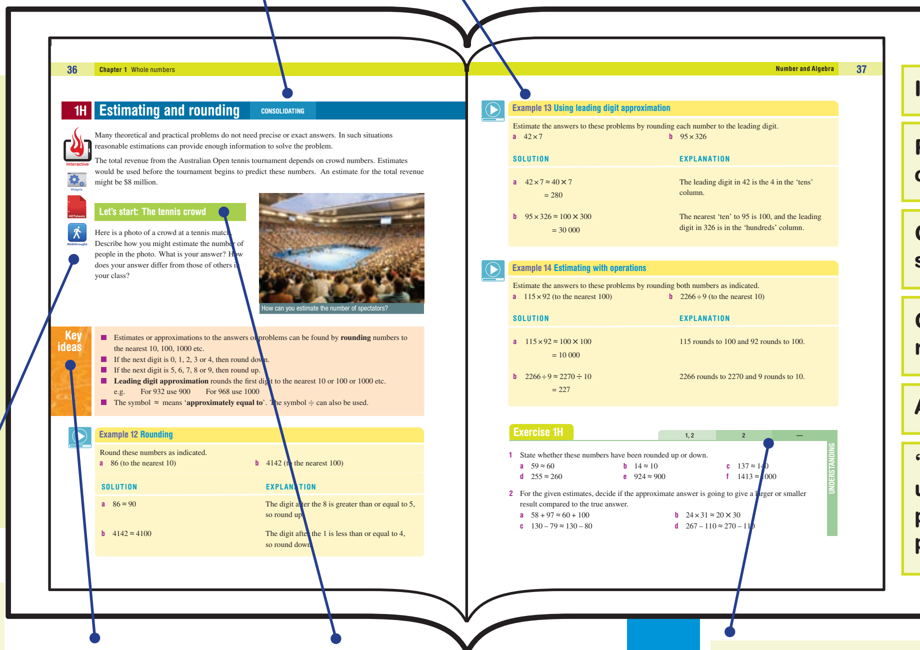
An overview of the *Essential Mathematics for the Australian Curriculum* complete learning suite

For more detail, see the guide in the online Interactive Textbook

PRINT TEXTBOOK

All non-core topics are marked Consolidating or Extending to assist with course planning

Examples with fully worked solutions and explanations show the thinking behind each step



- Investigations
- Problems and challenges
- Chapter summaries
- Chapter reviews
- Answers
- 'Working with unfamiliar problems' poster

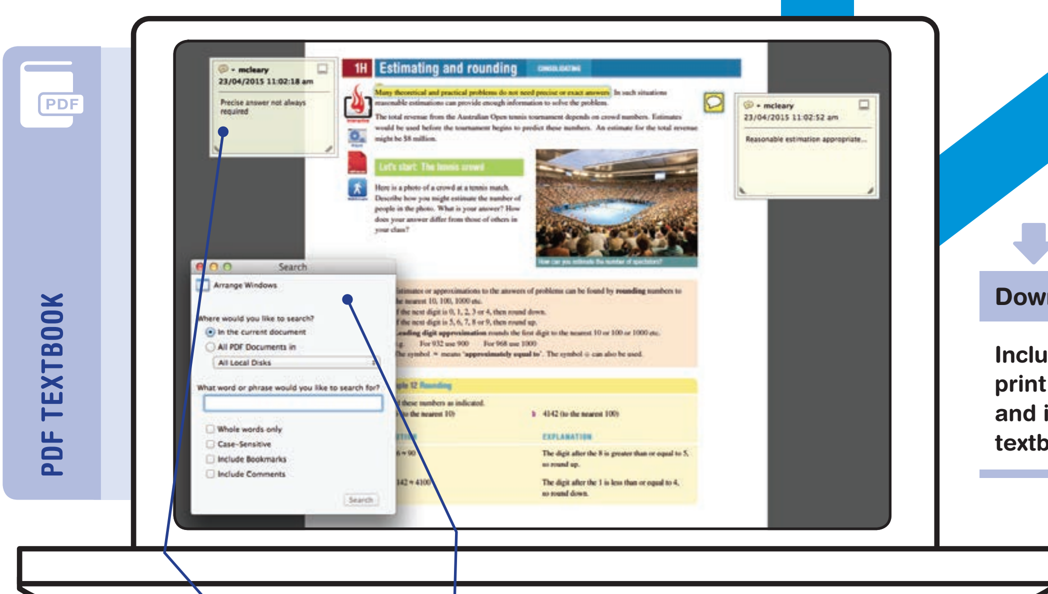
Each topic in the print book comes with interactive *HOTmaths* widgets, walkthroughs and HOTsheets in the interactive textbook

Key ideas summarise key knowledge and skills for each lesson

Let's start activities get students thinking critically and creatively about the topic

Working programs subtly embedded in each Proficiency Strand to provide three learning pathways through the book

PDF TEXTBOOK



Downloadable

Included with print textbook and interactive textbook

Note-taking

Search functions

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Easy navigation within sections without scrolling

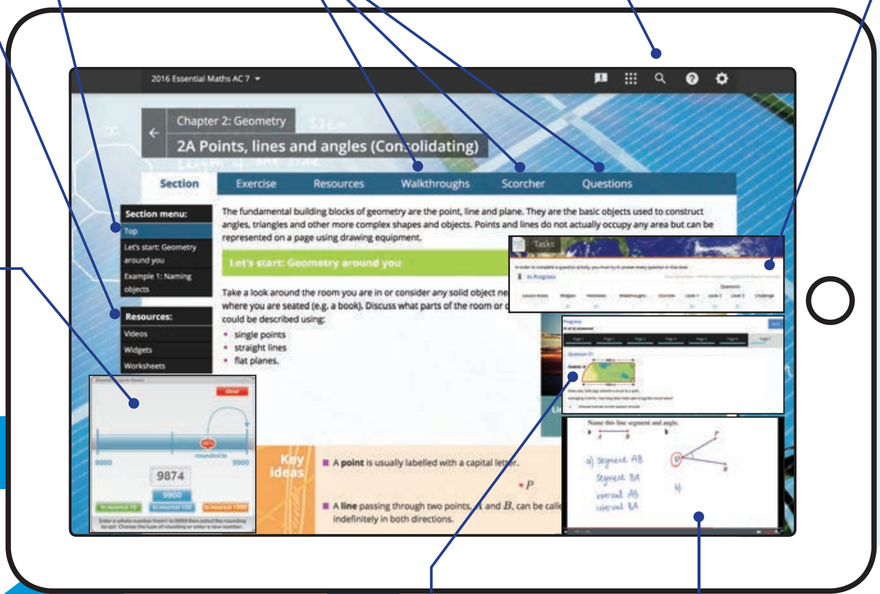
Walkthroughs, Scorcher and levelled question sets in every section

Interactive navigation and searching

Tasks sent by teacher

Access to interactive resources at any time without leaving the page

Hundreds of interactive widgets



INTERACTIVE TEXTBOOK POWERED BY *HOTmaths*

Online tests sent by teacher

Video demonstration for every worked example

Student reporting

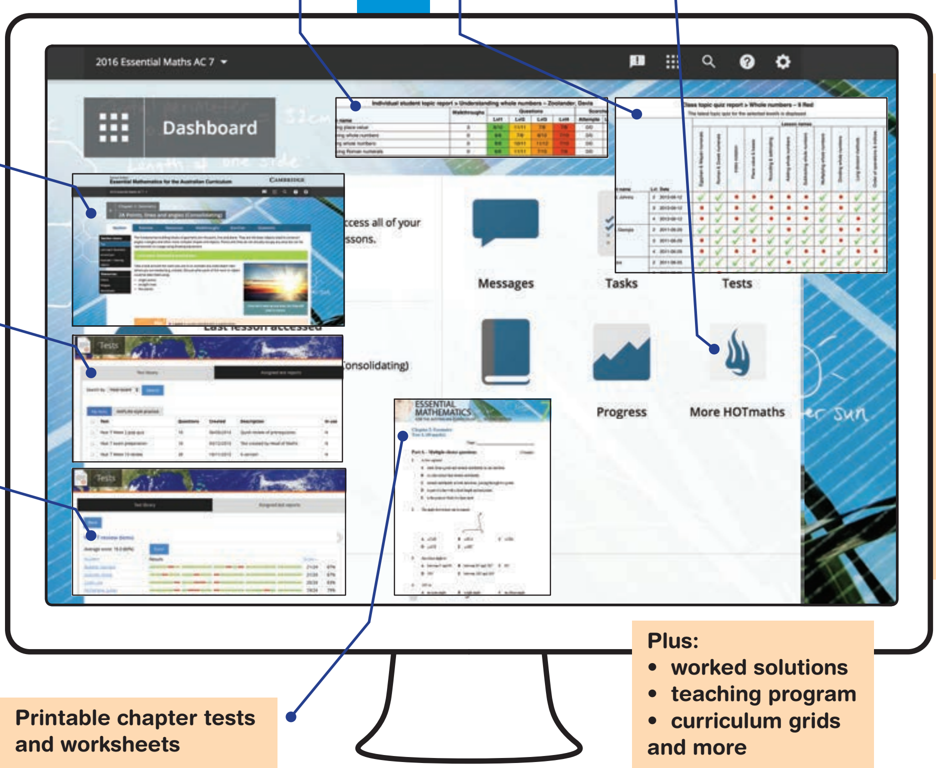
Class reporting

Access to all *HOTmaths* Australian Curriculum courses

Teacher's copy of interactive textbook

Test generator and ready-made tests

Student results



ONLINE TEACHING SUITE POWERED BY *HOTmaths*

Printable chapter tests and worksheets

Plus:
- worked solutions
- teaching program
- curriculum grids

and more

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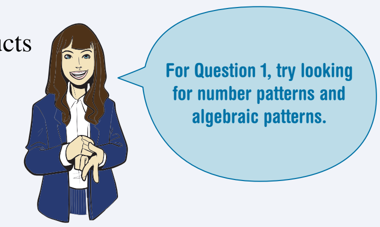
Working with unfamiliar problems: Part 1

The questions on the next four pages are designed to provide practice in solving unfamiliar problems. Use the 'Working with unfamiliar problems' poster at the back of this book to help you if you get stuck.

In Part 1, apply the suggested strategy to solve these problems, which are in no particular order. Clearly communicate your solution and final answer.



1 Discover the link between Pascal's triangle and expanded binomial products and use this pattern to help you expand $(x + y)^6$.

Pascal's triangle

$(x + y)^0$	1
$(x + y)^1$	1 1
$(x + y)^2$	1 2 1
$(x + y)^3$	1 3 3 1

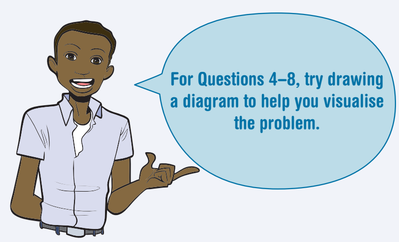
2 How many palindromic numbers are there between 10^1 and 10^3?

3 Find the smallest positive integer values for x so that $60x$ is:

- **i** a perfect square
- **ii** a perfect cube
- **iii** divisible by both 8 and 9.

4 A Year 10 class raises money at a fete by charging players $1 to flip their dollar coin onto a red and white checked tablecloth with 50 mm squares. If the dollar coin lands fully inside a red square the player keeps their $1. What is the probability of keeping the $1? How much cash is likely to be raised from 64 players?

25 mm



5 The shortest side of a 60° set square is 12 cm. What is the length of the longest side of this set square?

6 A Ferris wheel with diameter 24 metres rotates at a constant rate of 60 seconds per revolution.

- **a** Calculate the time taken for a rider to travel:
  - **i** from the bottom of the wheel to 8 m vertically above the bottom
  - **ii** from 8 m to 16 m vertically above the bottom of the wheel.
- **b** What fraction of the diameter is the vertical height increase after each one-third of the ride from the bottom to the top of the Ferris wheel?

7 $ABCD$ is a rectangle with $AB = 16$ cm and $AD = 12$ cm. X and Y are points on BD such that AX and CY are each perpendicular to the diagonal BD. Find the length of the interval XY.

8 How many diagonal lines can be drawn inside a decagon (i.e. a 10-sided polygon)?

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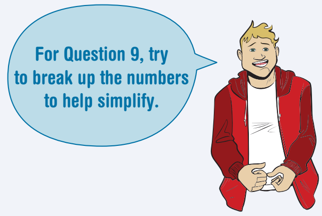
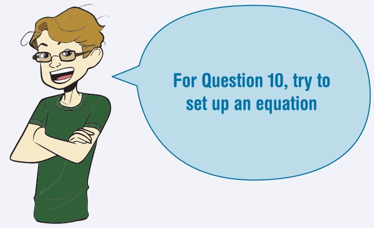
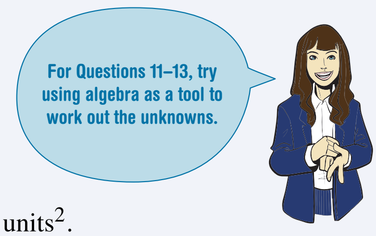
9 The symbol ! means factorial.
e.g. $4! = 4 \times 3 \times 2 \times 1 = 24$.
Simplify $9! \div 7!$ without the use of a calculator.





10 In 2017 Charlie's age is the sum of the digits of his birth year $19xy$ and Bob's age is one less than triple the sum of the digits of his birth year $19yx$. Find Charlie's age and Bob's age on their birthdays in 2017.

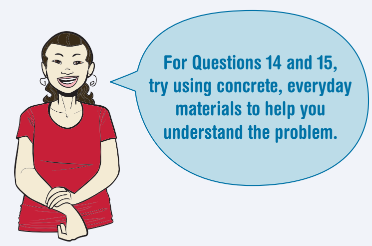
11 Let D be the difference between the squares of two consecutive positive integers. Find an expression for the average of the two integers in terms of D.



12 For what value of b is the expression $15ab + 6b - 20a - 8$ equal to zero for all values of a?

13 Find the value of k given $k > 0$ and that the area enclosed by the lines $y = x + 3$, $x + y + 5 = 0$, $x = k$ and the y-axis is 209 units^2.

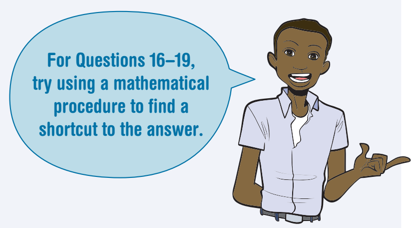
14 The diagonal of a cube is $\sqrt{27}$ cm. Calculate the volume and surface area of this cube.



15 Two sides of a triangle have lengths 8 cm and 12 cm, respectively. Determine between which two values the length of the third side would fall. Give reasons for your answer.

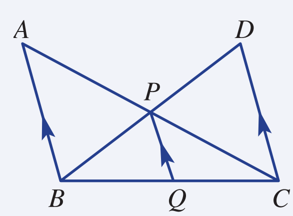
16 When $10^{89} - 89$ is expressed as a single number, what is the sum of its digits?

17 Determine the reciprocal of this product:
$(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \ldots (1 - \frac{1}{n+1})$.



18 Find the value of $\dfrac{1002^2 - 998^2}{102^2 - 98^2}$, without using a calculator.

19 In the diagram at right, $AP = 9$ cm, $PC = 15$ cm, $BQ = 8.4$ cm and $QC = 14$ cm. Also, $CD \parallel QP \parallel BA$. Determine the ratio of the sides AB to DC.



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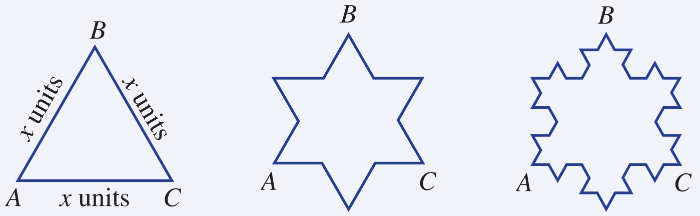
Working with unfamiliar problems: Part 2

For the questions in Part 2, again use the 'Working with unfamiliar problems' poster at the back of this book, but this time choose your own strategy (or strategies) to solve each problem. Clearly communicate your solution and final answer.

1 The Koch snowflake design starts with an equilateral triangle. A smaller equilateral triangle is built onto the middle third of each side and its base is erased. This procedure can be repeated indefinitely.



a For a Koch snowflake with initial triangle side length x units, determine expressions for the exact value of:

i the perimeter after 5 procedure repeats and after n procedure repeats

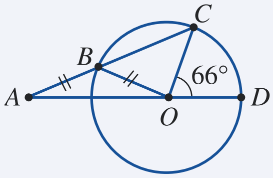
ii the sum of areas after 3 procedure repeats and the *change* in area after n procedure repeats.

b Comment on perimeter and area values as $n \to \infty$. Give reasons for your answers.

2 Two sides of a triangle have lengths in the ratio 3 : 5 and the third side has length 37 cm. If each side length has an integer value, find the smallest and largest possible perimeters, in cm.

3 The midpoints of each side of a regular hexagon are joined to form a smaller regular hexagon with side length k cm. Determine a simplified expression in terms of k for the exact difference in the perimeters of the two hexagons.

4 Angle COD is 66°. Find the size of angle CAD.



5 The graph of $y = ax^2 + 2x + 3$ has an axis of symmetry at $x = \frac{1}{4}$. Determine the maximum possible value of y.

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6 Find the value of x and y given that $5^x = 125^{y-3}$ and $81^{x+1} = 9^y \times 3$.

7 A rectangular prism has a surface area of 96 cm^2 and the sum of the lengths of all its edges is 64 cm. Determine the exact sum of the lengths of all its internal diagonals (i.e. diagonals not on a face).

8 In a Year 10 maths test, six students gained 100%, all students scored at least 75% and the mean mark was 82.85%. If the results were all whole numbers, what is the smallest possible number of students in this class? List the set of results for this class size.

9 Determine the exact maximum vertical height of the line $y = 2x$ above the parabola $y = 2x^2 - 5x - 3$.

10 $A + B = 6$ and $AB = 4$. Without solving for A and B, determine the values of:

a $(A + 1)(B + 1)$ **b** $A^2 + B^2$ **c** $(A - B)^2$ **d** $\frac{1}{A} + \frac{1}{B}$

11 If $f(1) = 5$ and $f(x + 1) = 2f(x)$, determine the value of $f(8)$.

12 Four rogaining markers, $PQRS$, are in an area of bushland with level ground. Q is 1.4 km east of P, S is 1 km from P on a true bearing of 168° and R is 1.4 km from Q on a true bearing of 200°. To avoid swamps, Lucas runs the route $PRSQP$. Calculate the distances (in metres) and the true bearings from P to R, from R to S, from S to Q and from Q to P. Round your answers to the nearest whole number.

13 Consider all points (x, y) that are equidistant from the point (4, 1) and the line $y = -3$. Find the rule relating x and y and then sketch its graph, labelling all significant features. (Note: Use the distance formula)

14 A 'rule of thumb' useful for 4WD beach driving is that the proportion of total tide height change after either high or low tide is $\frac{1}{12}$ in the first hour, $\frac{2}{12}$ in the second hour, $\frac{3}{12}$ in the third hour, $\frac{3}{12}$ in the fourth hour, $\frac{2}{12}$ in the fifth hour and $\frac{1}{12}$ in the sixth hour.

a Determine the accuracy of this 'rule of thumb' using the following equation for tide height: $h = 0.7\cos(30t) + 1$, where h is in metres and t is time in hours after high tide.

b Using $h = A\cos(30t) + D$, show that the proportion of total tide height change between any two given times t_1 and t_2 is independent of the values of A and D.

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Chapter 1 Linear relations

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Patterns and algebra

Factorise algebraic expressions by taking out a common algebraic factor.

Apply the four operations to simple algebraic fractions with numerical denominators.

Substitute values into formulas to determine an unknown.

Linear and non-linear relationships

Solve problems involving linear equations, including those derived from formulas.

Solve linear inequalities and graph their solutions on a number line.

Solve linear simultaneous equations, using algebraic and graphical techniques, including using digital technology.

Solve problems involving parallel and perpendicular lines.

Solve linear equations involving simple algebraic fractions.

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Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Iron ore mining

The revenue and costs of mining iron ore for Australian mining companies depends on many factors, including the quality of iron ore, the number of qualified staff, the amount of equipment, the price of iron and the depth of the iron ore in the ground. Maximising profit therefore requires the need to balance all the cost factors against the revenue.

This is achieved by using a mathematical process called linear programming, which involves graphing all the various constraints as straight lines to create a feasible region on an *x*–*y* plane. The profit equation is then graphed over this feasible region to determine how maximum profit can be achieved. Such simple mathematical analysis can save mining companies millions of dollars.

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1A Reviewing algebra CONSOLIDATING

Interactive Widgets HOTsheets Walkthroughs

Algebra involves the use of pronumerals (or variables), which are letters representing numbers. Combinations of numbers and pronumerals form terms (numbers and pronumerals connected by multiplication and division), expressions (a term or terms connected by addition and subtraction) and equations (mathematical statements that include an equals sign). Skills in algebra are important when dealing with the precise and concise nature of mathematics. The complex nature of many problems in finance and engineering usually result in algebraic expressions and equations that need to be simplified and solved.


Stockmarket traders rely on financial modelling based on complex algebraic expressions.

Let's start: Mystery problem

Between one school day and the next, the following problem appeared on a student noticeboard.

Prove that $8 - x^2 + \frac{3x - 9}{3} + 5(x - 1) - x(6 - x) = 0$.

- By working with the left-hand side of the equation, show that this equation is true for any value of x.
- At each step of your working, discuss what algebraic processes you have used.

Key ideas

- Algebra uses the following words:
  - **term:** $5x$, $7x^2y$, $\frac{2a}{3}$, 7 (a constant term)
  - **coefficient:** -3 is the coefficient of x^2 in $7 - 3x^2$; 1 is the coefficient of y in $y + 7x$.
  - **expression:** $7x$, $3x + 2xy$, $\frac{x + 3}{2}$
  - **equation:** $x = 5$, $7x - 1 = 2$, $x^2 + 2x = -4$
- Expressions can be evaluated by substituting a value for each pronumeral (variable).
  - Order of operations are followed: First brackets, then indices, then multiplication and division, then addition and subtraction, working then from left to right.

    If $x = -2$ and $y = 4$, then

$$\frac{3x^2 - y}{2} = \frac{3(-2)^2 - 4}{2} = \frac{3 \times 4 - 4}{2} = 4$$

- **Like terms** have the same pronumeral part and, using addition and subtraction, can be collected to form a single term.

  For example, $3x - 7x + x = -3x$

  $6a^2b - ba^2 = 5a^2b$

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Key ideas

- The symbols for multiplication (×) and division (÷) are usually not shown.

$$7 \times x \div y = \frac{7x}{y}$$

$$-6a^2b \div (ab) = \frac{-6a^2b}{ab}$$

$$= -6a$$

- The **distributive law** is used to expand brackets.
  - $a(b + c) = ab + ac$ $\quad 2(x + 7) = 2x + 14$
  - $a(b - c) = ab - ac$ $\quad -x(3 - x) = -3x + x^2$

- **Factorisation** involves writing expressions as a product of factors.
  - Many expressions can be factorised by taking out the highest common factor (HCF).
    $15 = 3 \times 5$
    $3x - 12 = 3(x - 4)$
    $9x^2y - 6xy + 3x = 3x(3xy - 2y + 1)$

- Other general properties are:
  - **associative** $\quad a \times (b \times c) = (a \times b) \times c$ or $a + (b + c) = (a + b) + c$
  - **commutative** $\quad ab = ba$ or $a + b = b + a$ $\left(\text{Note: } \frac{a}{b} \neq \frac{b}{a} \text{ and } a - b \neq b - a.\right)$
  - **identity** $\quad a \times 1 = a$ or $a + 0 = a$
  - **inverse** $\quad a \times \frac{1}{a} = 1$ or $a + (-a) = 0$

Example 1 Collecting like terms

Simplify by collecting like terms.

a $3a^2b - 2a^2b$

b $5xy + 2xy^2 - 2xy + 3y^2x$

SOLUTION	EXPLANATION
a $3a^2b - 2a^2b = a^2b$	$3a^2b$ and $2a^2b$ have the same pronumeral part, so they are like terms. Subtract coefficients and recall that $1a^2b = a^2b$.
b $5xy + 2xy^2 - 2xy + 3y^2x = 3xy + 5xy^2$	Collect like terms, noting that $3y^2x = 3xy^2$. The + or – sign belongs to the term that directly follows it.

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Example 2 Multiplying and dividing expressions

Simplify the following.

a $2h \times 7l$ **b** $-3p^2r \times 2pr$ **c** $-\frac{7xy}{14y}$

SOLUTION

EXPLANATION

a $2h \times 7l = 14hl$

Multiply the numbers and remove the × sign.

b $-3p^2r \times 2pr = -6p^3r^2$

Remember the basic index law: When you multiply terms with the same base you add the powers.

c $-\frac{7xy}{14y} = -\frac{x}{2}$

Cancel the highest common factor of 7 and 14 and cancel the y.

Example 3 Expanding the brackets

Expand the following using the distributive law. Simplify where possible.

a $2(x+4)$ **b** $-3x(x-y)$ **c** $3(x+2)-4(2x-4)$

SOLUTION

EXPLANATION

a $2(x+4) = 2x+8$

$2(x+4) = 2 \times x + 2 \times 4$

b $-3x(x-y) = -3x^2 + 3xy$

Note that $x \times x = x^2$ and $-3 \times (-1) = 3$.

c $3(x+2)-4(2x-4) = 3x+6-8x+16$

$= -5x+22$

Expand each pair of brackets and simplify by collecting like terms.

Example 4 Factorising simple algebraic expressions

Factorise:

a $3x-9$ **b** $2x^2+4x$

SOLUTION

EXPLANATION

a $3x-9 = 3(x-3)$

HCF of $3x$ and 9 is 3.

Check that $3(x-3) = 3x-9$.

b $2x^2+4x = 2x(x+2)$

HCF of $2x^2$ and $4x$ is $2x$.

Check that $2x(x+2) = 2x^2+4x$.

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Example 5 Evaluating expressions

Evaluate $a^2 - 2bc$ if $a = -3$, $b = 5$ and $c = -1$.

SOLUTION	EXPLANATION
$a^2 - 2bc = (-3)^2 - 2(5)(-1)$	Substitute for each pronumeral:
$= 9 - (-10)$	$(-3)^2 = -3 \times (-3)$ and $2 \times 5 \times (-1) = -10$
$= 19$	To subtract a negative number, add its opposite.

Exercise 1A

1, 2, 3(½), 4, 5	4, 5	—

UNDERSTANDING

1 Which of the following is an equation?

A $3x - 1$ B $\frac{x+1}{4}$ C $7x + 2 = 5$ D $3x^2y$

2 Which expression contains a term with a coefficient of –9?

A $8 + 9x$ B $2x + 9x^2y$ C $9a - 2ab$ D $b - 9a^2$

3 State the coefficient of a^2 in these expressions.

a $4 + 7a^2$ b $a + a^2$ c $\frac{3}{2} - 4a^2$ d $-9a^2 + 2a^3$

e $\frac{a^2}{2}$ f $1 - \frac{a^2}{5}$ g $\frac{2}{7}a^2 + a$ h $-\frac{7a^2}{3} - 1$

4 Decide whether the following pairs of terms are like terms.

a xy and $2yx$ b $7a^2b$ and $-7ba^2$ c $-4abc^2$ and $8ab^2c$

5 Evaluate:

a $(-3)^2$ b $(-2)^3$ c -2^3 d -3^2

6–11(½)	6–11(½)	6–11(½)

FLUENCY

Example 1

6 Simplify by collecting like terms.

a $6a + 4a$ b $8d + 7d$ c $5y - 5y$

d $2xy + 3xy$ e $9ab - 5ab$ f $4t + 3t + 2t$

g $7b - b + 3b$ h $3st^2 - 4st^2$ i $4m^2n - 7nm^2$

j $0.3a^2b - ba^2$ k $4gh + 5 - 2gh$ l $7xy + 5xy - 3y$

m $4a + 5b - a + 2b$ n $3jk - 4j + 5jk - 3j$ o $2ab^2 + 5a^2b - ab^2 + 5ba^2$

p $3mn - 7m^2n + 6nm^2 - mn$ q $4st + 3ts^2 + st - 4s^2t$ r $7x^3y^4 - 3xy^2 - 4y^4x^3 + 5y^2x$

Example 2

7 Simplify the following.

a $4a \times 3b$ b $5a \times 5b$ c $-2a \times 3d$ d $5h \times (-2m)$

e $-6h \times (-5t)$ f $-5b \times (-6l)$ g $2s^2 \times 6t$ h $-3b^2 \times 7d^5$

i $4ab \times 2ab^3$ j $-6p^2 \times (-4pq)$ k $6hi^4 \times (-3h^4i)$ l $7mp \times 9mr$

m $\frac{7x}{7}$ n $\frac{6ab}{2}$ o $-\frac{3a}{9}$ p $-\frac{2ab}{8}$

q $\frac{4ab}{2a}$ r $-\frac{15xy}{5y}$ s $-\frac{4xy}{8x}$ t $-\frac{28ab}{56b}$

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1A

FLUENCY

Example 3a,b

8 Expand the following, using the distributive law.

- **a** $5(x+1)$
- **b** $2(x+4)$
- **c** $3(x-5)$
- **d** $-5(4+b)$
- **e** $-2(y-3)$
- **f** $-7(a+c)$
- **g** $-6(-m-3)$
- **h** $4(m-3n+5)$
- **i** $-2(p-3q-2)$
- **j** $2x(x+5)$
- **k** $6a(a-4)$
- **l** $-4x(3x-4y)$
- **m** $3y(5y+z-8)$
- **n** $9g(4-2g-5h)$
- **o** $-2a(4b-7a+10)$
- **p** $7y(2y-2y^2-4)$
- **q** $-3a(2a^2-a-1)$
- **r** $-t(5t^3+6t^2+2)$
- **s** $2m(3m^3-m^2+5m)$
- **t** $-x(1-x^3)$

Example 3c

9 Expand and simplify the following, using the distributive law.

- **a** $2(x+4)+3(x+5)$
- **b** $4(a+2)+6(a+3)$
- **c** $6(3y+2)+3(y-3)$
- **d** $3(2m+3)+3(3m-1)$
- **e** $2(2+6b)-3(4b-2)$
- **f** $3(2t+3)-5(2-t)$
- **g** $2x(x+4)+x(x+7)$
- **h** $4(6z-4)-3(3z-3)$
- **i** $3d^2(2d^3-d)-2d(3d^4+4d^2)$
- **j** $q^3(2q-5)+q^2(7q^2-4q)$

Example 4

10 Factorise:

- **a** $3x-9$
- **b** $4x-8$
- **c** $10y+20$
- **d** $6y+30$
- **e** x^2+7x
- **f** $2a^2+8a$
- **g** $5x^2-5x$
- **h** $9y^2-63y$
- **i** $xy-xy^2$
- **j** $x^2y-4x^2y^2$
- **k** $8a^2b+40a^2$
- **l** $7a^2b+ab$
- **m** $-5t^2-5t$
- **n** $-6mn-18mn^2$
- **o** $-y^2-8yz$
- **p** $-3a^2b-6ab-3a$

Example 5

11 Evaluate these expressions if $a=-4$, $b=3$ and $c=-5$.

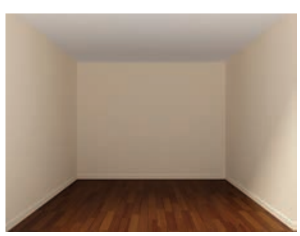
- **a** $-2a^2$
- **b** $b-a$
- **c** $abc+1$
- **d** $-ab$
- **e** $\frac{a+b}{2}$
- **f** $\frac{3b-a}{5}$
- **g** $\frac{a^2-b^2}{c}$
- **h** $\frac{\sqrt{a^2+b^2}}{\sqrt{c^2}}$

12	12, 13	12, 13

PROBLEM-SOLVING

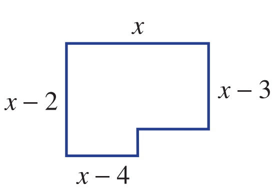
12 Find an expression for the area of a floor of a rectangular room with the following side lengths. Expand and simplify your answer.

- **a** $x+3$ and $2x$
- **b** x and $x-5$



13 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. Note: All angles are right angles.

a



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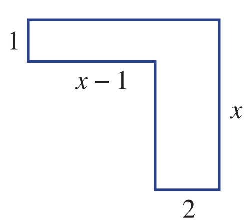

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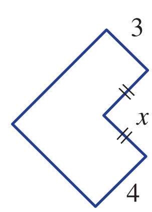

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REASONING

14, 15(½)	14, 15	15(½), 16

14 When $a = -2$ give reasons why:

a $a^2 > 0$ **b** $-a^2 < 0$ **c** $a^3 < 0$

15 Decide whether the following are true or false for all values of a and b. If false, give an example to show that it is false.

a $a + b = b + a$
b $a - b = b - a$
c $ab = ba$
d $\frac{a}{b} = \frac{b}{a}$
e $a + (b + c) = (a + b) + c$
f $a - (b - c) = (a - b) - c$
g $a \times (b \times c) = (a \times b) \times c$
h $a \div (b \div c) = (a \div b) \div c$

16 **a** Write an expression for the statement 'the sum of x and y divided by 2'.
b Explain why the statement above is ambiguous.
c Write an unambiguous statement describing $\frac{a+b}{2}$.

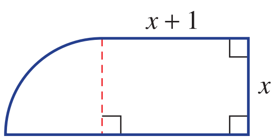
Algebraic circular spaces

ENRICHMENT

—	—	17

17 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes. Your answers may contain π, for example 4π. Do not use decimals.

a



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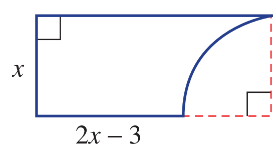
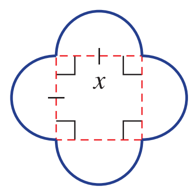


c



Architects, builders, carpenters and landscapers are among the many occupations that use algebraic formulas to calculate areas and perimeters in daily work.

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